Recent progress in particle acceleration theory

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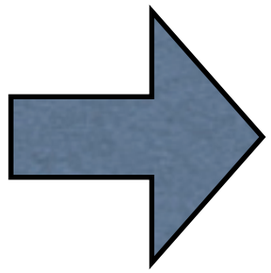
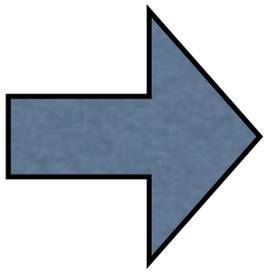
Two sources of free energy for charged particle acceleration:

- Differential motion of magnetised plasma



Fermi mechanisms

- Magnetic field energy



Magnetic reconnection

(of course magnetic energy usually comes from differential motion also!)

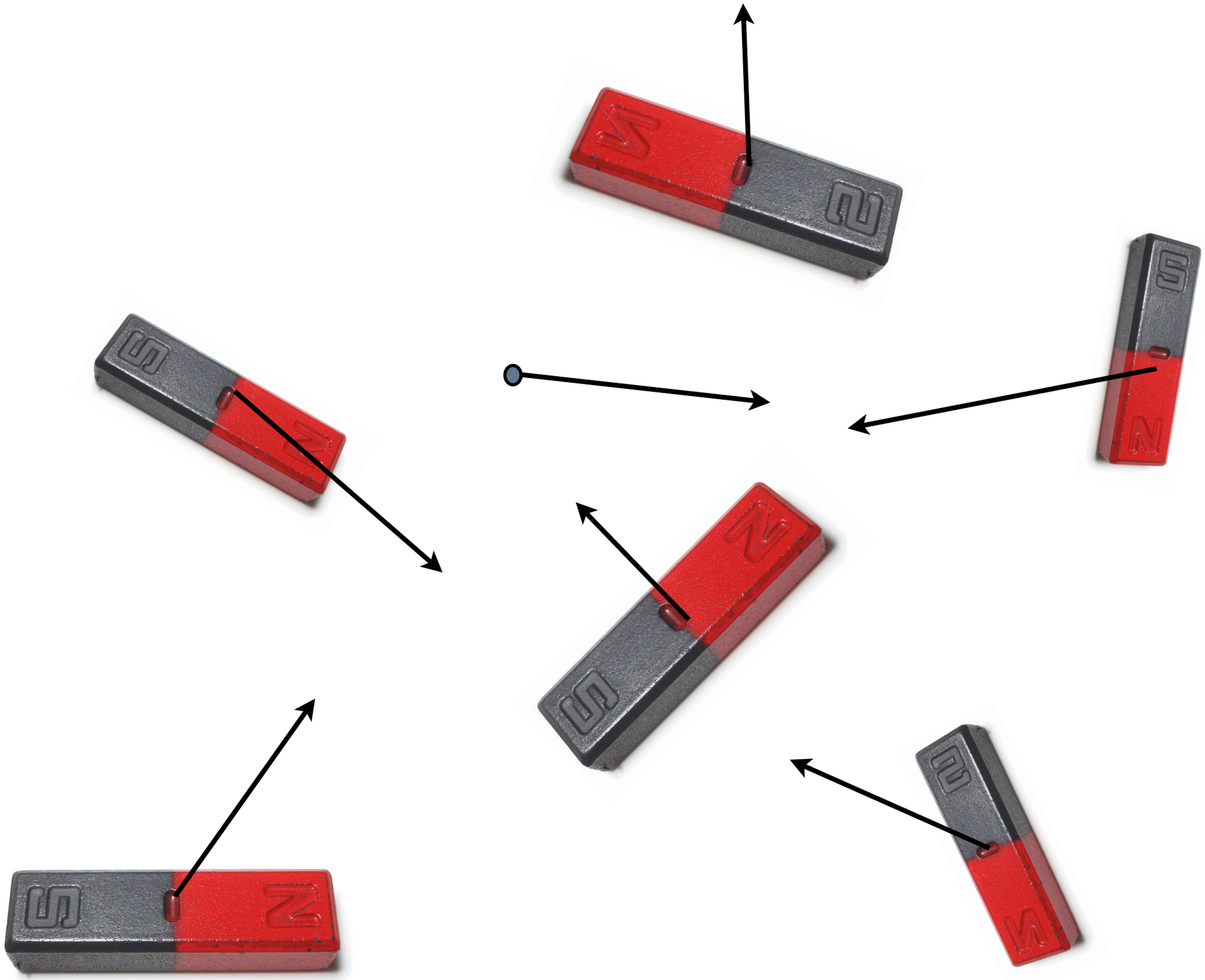
Theory has mainly concentrated on Fermi processes because:

- they seem to be more important in most systems;
- we have a well-developed theory and can, at least in principle, calculate models.

Reconnection should not be forgotten however!
Works in the sun, Earth's magnetotail and probably pulsars. However for rest of talk will concentrate on Fermi processes.

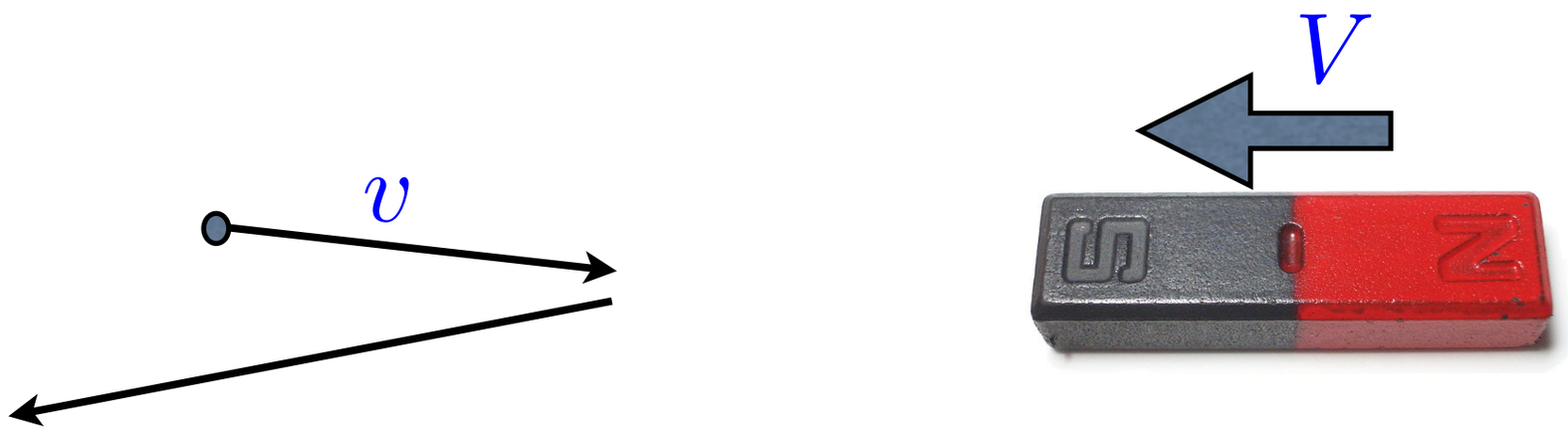
Fermi's insight

- Magnetic fields couple microscopic degrees of freedom of individual charged particles to macroscopic bulk motion of plasma.
- Attempt to achieve equilibrium inevitably leads, on average, to energy transfer to particles.
- *Gedanken* experiment - think of "gas" of bar magnets plus one protons....



$$\frac{1}{2}m_p v^2 \rightarrow \frac{1}{2}MV^2$$

`Head on' collisions give an energy (or momentum) gain of order



$$\frac{\Delta p}{p} \approx \frac{V}{v}$$

NB importance of elastic scattering off very heavy magnetic mirror!

- But if collisions are random this merely gives a random walk (diffusion) of the charged particles in phase space (second order Fermi).

- Very slow normally, $t_{\rm acc} \approx \left(\frac{c}{V}\right)^2 t_{\rm coll}$ especially if $V \approx v_{\rm Alfven}$

- Big discovery in 1977 - fast and efficient version of Fermi acceleration associated with shock waves (first order Fermi).

⇒ Diffusive Shock Acceleration

- Note that if the differential plasma motions are relativistic, second order and first order become comparable!

- For first order need plasma compression, effect can be partially undone by subsequent re-expansion.

- Ensemble of weak shocks is equivalent to second order process.

- Shocks are generic features of nonlinear hyperbolic systems such as MHD. Increase of signal speed on compression steepens compressive regions of flow.

- Physically appear as self-organising quasi-one-dimensional dissipative structures in which KE of bulk motion is transferred to micro-scale random motion.

- Dissipation in collisionless shock comes from collective plasma processes (not, as in gas dynamics, from 2-body collisions).

Diffusive Shock Acceleration

- Belongs to the general class of Fermi acceleration processes - energy comes from differential motion in magnetised plasma (here velocity jump in shock)
- Very efficient because collisionless plasma shocks are highly compressive and have strongly disordered magnetic fields (thus distributions close to isotropy, transport is diffusive)

Issues

- Nonlinear modifications
- Magnetic field effects
- Relativistic effects
- Plasma physics of the subshock
- Second order Fermi terms

In reality all coupled!

Nonlinear Reaction

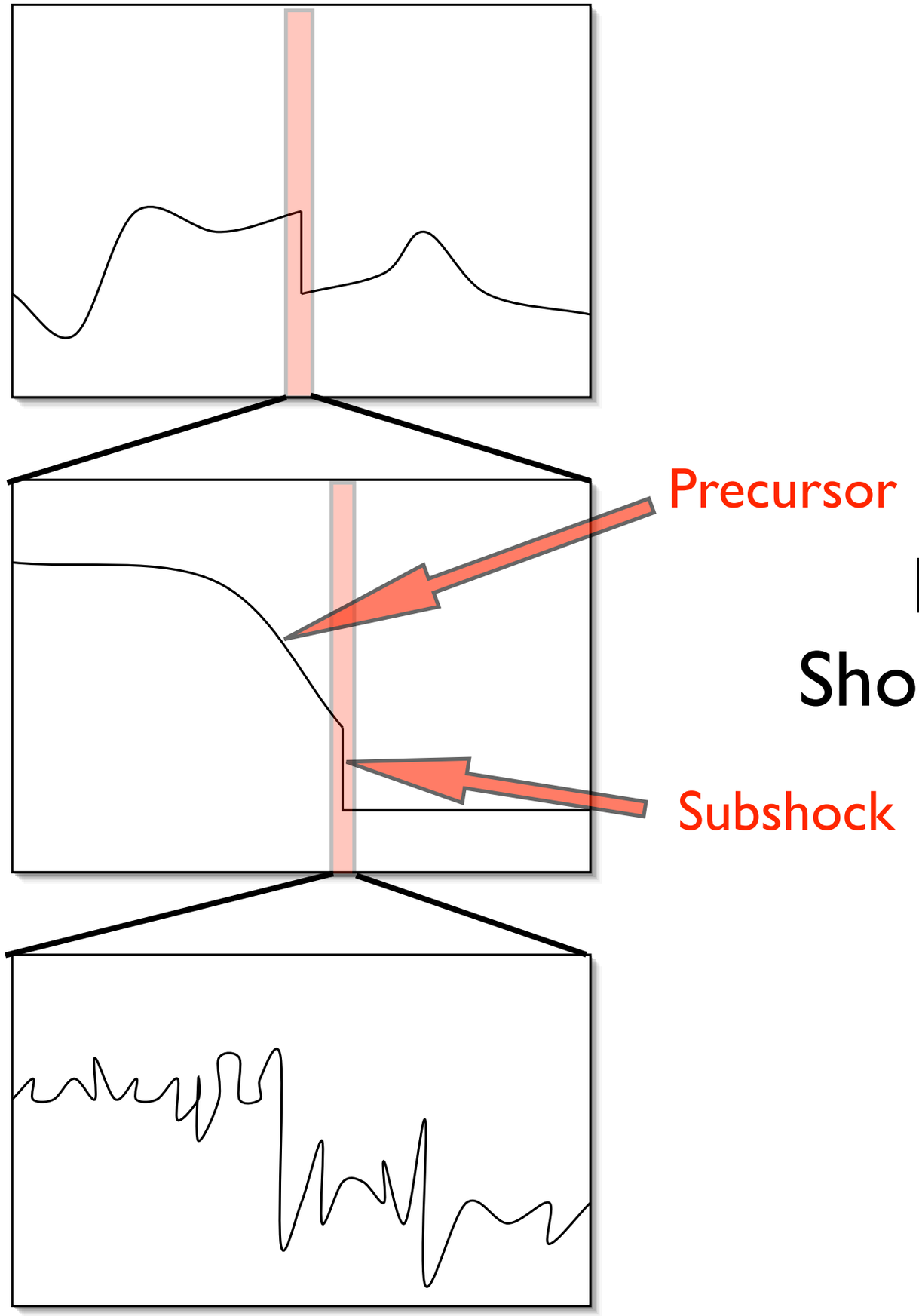
- The "big issue" for first twenty years of DSA
- Now regarded as "solved" (but with caveats)
- Shock structure has to be modified if the accelerated particles take a significant part of the energy dissipated in the shock (as has to be the case for many models)

In principle easy - we just have to solve the diffusive transport equation and the usual hydrodynamic equations with an additional cosmic ray pressure in the momentum equation!

$$P_C(x) = \int \frac{4\pi p^3 v}{3} f(p,x)\, dp$$

$$\frac{\partial \rho \vec{U}}{\partial t} + \nabla \cdot \left(\rho \vec{U} \otimes \vec{U} + P_G + P_C \right) = 0$$

NB this does assume strong coupling of the accelerated particles (cosmic rays) to the background through scattering via the magnetic field.



Outer scale
Astrophysics

Intermediate scales
Shock acceleration theory

Inner scale
Plasma physics
Injection!

Shock modification

- Extended upstream precursor + subshock structure
- Increased total compression due to
  - softer equation of state
  - additional energy flux to high energy particles (escape, geometrical dilution, diffusion)

- Spectrum at low energies given by test-particle theory applied to the sub-shock, thus softer.
- Spectrum at high energies should reflect much increased compression of total shock structure, thus harder.
- Concave spectrum - no longer perfect power-law.

Semi-analytic approach to steady mesoscopic structure

Can (hopefully) assume steady planar structure with fixed mass and momentum fluxes.

$$\begin{aligned} \rho U &= A \\ AU + P_G + P_C &= B \end{aligned}$$

and we still have the steady balance between acceleration and loss downstream...

$$\frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$

.. but the problem is that the acceleration flux now depends on the upstream velocity profile *and* the particle distribution.

However, if one makes an *Ansatz*

$$f_0(p) \rightarrow f(x,p)$$

the particle conservation equation and the momentum balance equations, become two coupled equations (in general integro-differential) for the two unknown functions.

$$U(x), \quad f_0(p)$$

An obvious *Ansatz* would be to assume a distribution similar to that familiar from the test-particle theory,

$$f(x,p) = f_0(p) \exp \int \frac{U(x)\,dx}{\kappa(x,p)}$$

This is actually close to Malkov's *Ansatz* who, however, uses

$$f(x,p) = f_0(p) \exp \int \left(-\frac{1}{3} \frac{\partial \ln f_0}{\partial \ln p} \right) \frac{U(x)\,dx}{\kappa(x,p)}$$

which is asymptotically exact for strongly modified shocks.

Motivation comes from exact solution for uniformly distributed compression, *ie* linear velocity field. Easy to check that

$$f = p^{-7/2} \exp\left(\frac{-\alpha x^2}{2\kappa}\right)$$

identically satisfies

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial x} - \frac{1}{3}\frac{\partial U}{\partial x} p \frac{\partial f}{\partial p} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial f}{\partial x}\right)$$

for $U(x) = -x, \qquad \kappa \propto p, \qquad \alpha = \frac{7}{6}$

$$
\begin{aligned}
f(x,p) &= p^{-7/2}\exp\left(\frac{-\alpha x^2}{2\kappa}\right) \\
&= f_0(p)\int\left(-\frac{1}{3}\frac{\partial \ln f_o}{\partial \ln p}\right)\frac{U(x)\,dx}{\kappa}
\end{aligned}
$$

So the additional factor introduced by Malkov in the exponential can be thought of as compensating for the fact that the acceleration is distributed over the whole precursor and is not just concentrated at one point.

Blasi, Amato and Capriolli (2007) introduced a further factor to interpolate between these and recommend the following modified version of Malkov's *Ansatz*

$$f(x,p) = f_0(p) \exp \int \left(-\frac{1}{3} \frac{\partial \ln f_0}{\partial \ln p} \right) \left(1 - \frac{1}{r_{\rm sub}} \right) \frac{U(x)\, dx}{\kappa(x,p)}$$

A more physically motivated heuristic approach is to note that particles spend of order one diffusion time being accelerated by the precursor compression

$$\Delta p \approx \frac{t_{\rm diff}}{2} \frac{1}{3} (-\nabla \cdot U) p \approx \frac{1}{6} \frac{L}{U} \frac{\Delta U}{L} p = \frac{1}{6} \frac{\Delta U}{U} p$$

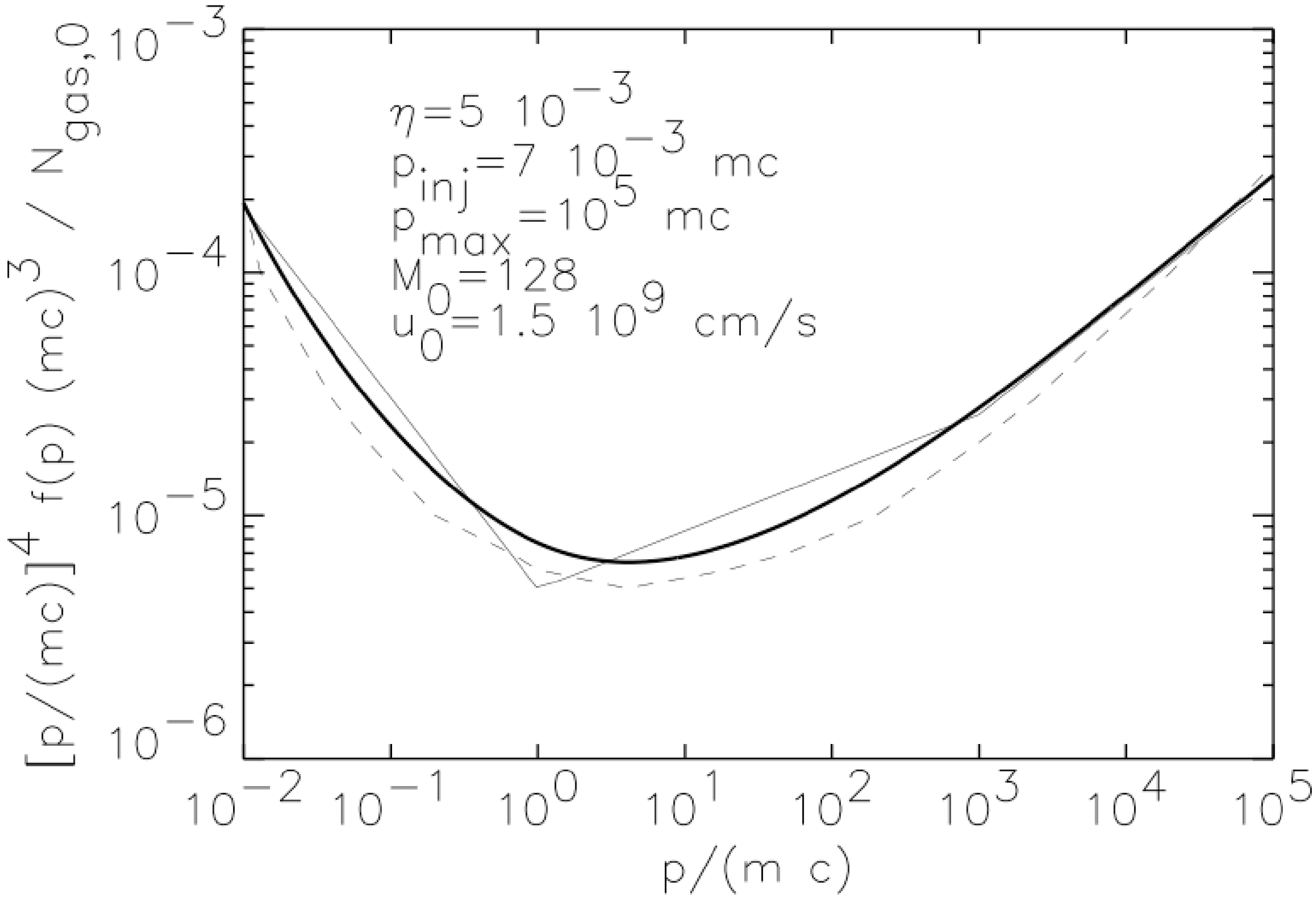
and thus they diffuse with an effective diffusion coefficient which is

$$\kappa - \Delta p \frac{\partial \kappa}{\partial p} \approx \kappa \left(1 - \frac{1}{6} \frac{\Delta U}{U} \frac{p}{\kappa} \frac{\partial \kappa}{\partial p} \right) .$$

This suggests the Ansatz

$$f(x,p) = f_0(p) \exp \int \left(1 + \frac{1}{6} \frac{U_0 - U_1}{U_0} \frac{\partial \ln \kappa}{\partial \ln p} \right) \frac{U(x)\, dx}{\kappa(x,p)}$$

which may work slightly better, but results are virtually identical on all Ansaetze.



From P. Blasi, 2002 - also agree very well with MC and full calculation

Consensus view...

- Spectra are generically curved, softer at low energies, hardening in the relativistic region before cutting off.
- Hardening at high energies at most changes spectral index from 4 to 3.5, so not too extreme
- Subshock is reduced to point where injection matches capacity of shock to accelerate; suggests minimum subshock compression ratio of about 2.5.

But...

- All approaches assume steady structure on the mesoscopic scale.
- In fact exist many possible instabilities.
- However can hope that theory still applies in mean sense - basic physics is very robust.
- Also not all bad news - offers exciting prospect of amplified B fields and thereby reaching higher energies.

The Instability Zoo

- Streaming excitation of Alfven waves (eg Wentzel, 1974; Skilling 1975)
- Acoustic instability (Drury and Falle, 1986)
- Parker instability (1966, 1967)
- McKenzie and Voelk, 1981 - wave heating or "plastic deformation of field".
- Bell and Lucek, 2000, 2001; Bell 2004, 2005
- Generic Weibel-type instabilities
- Jokipii - downstream vorticity

- Streaming excitation of Alfven waves
  - Originally proposed to give enhanced scattering at the shock (Bell 1978)
  - resonant v non-resonant terms (Achterberg, 1983; Bell, 2004; Reville 2006)
  - physical ordering of terms inappropriate for shock precursor case as noted by Bell.

$$\dot{E}_W \approx V_A \nabla P_C$$

- Acoustic instability
  - Exists in purely 1-D models (and thus important for numerical codes where it is always artificially supressed).
  - Depends on collective nature of scattering.

For small scale perturbations in 1-D

$$\kappa \nabla P_C \approx const$$

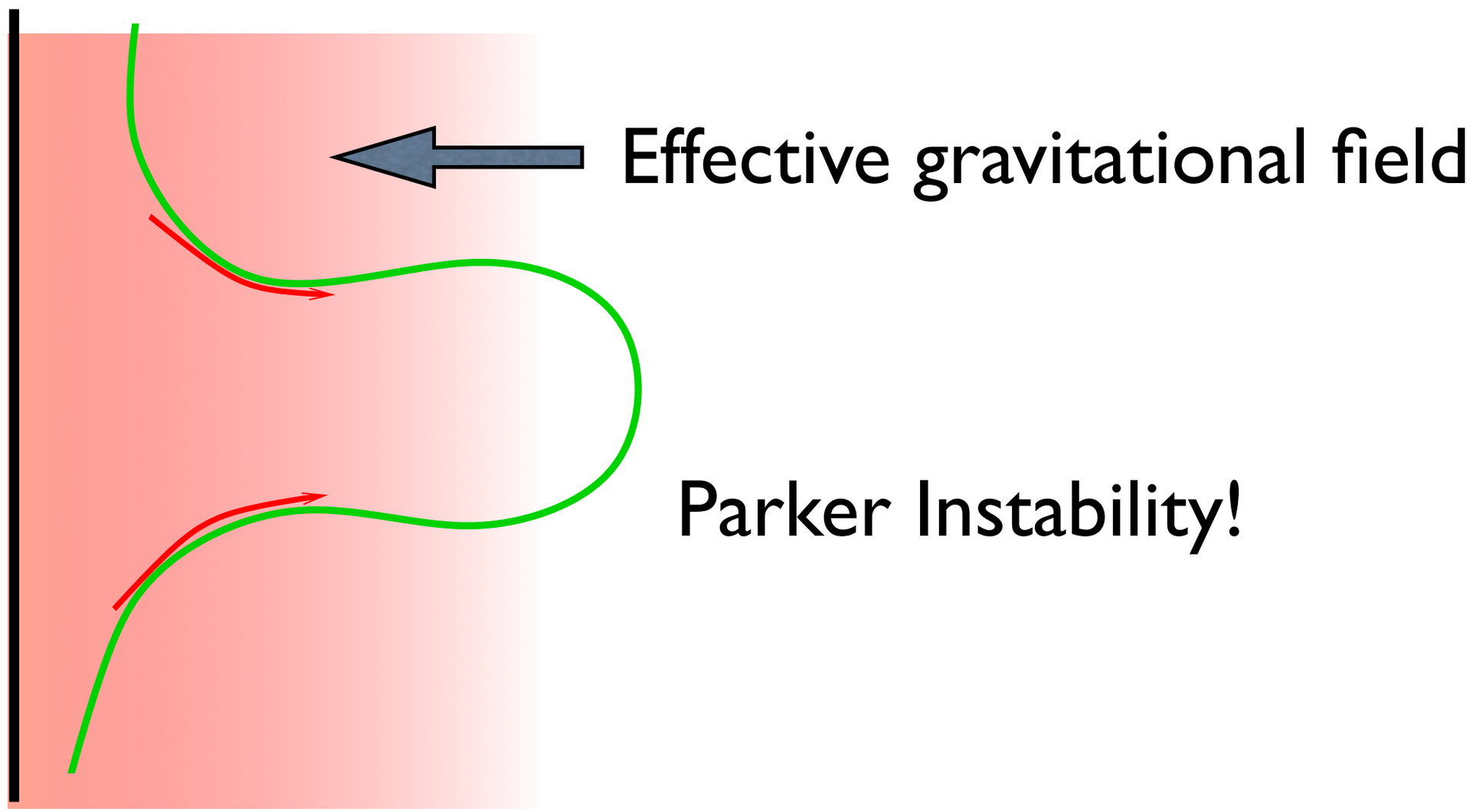
(drive constant flux through structures)

Associated acceleration fluctuations are

$$\frac{d^2x}{dt^2} \propto \frac{\nabla P_C}{\rho} \propto \frac{1}{\kappa\rho}$$

Unless the diffusion is exactly inversely proportional to the density (two body scattering case!) density fluctuations induce velocity fluctuation which rapidly amplify the initial perturbation. Mathematically appears as a mechanism driving sound waves unstable.

But in some sense not very physical as clearly relies strongly on the 1-D nature of the initial perturbation. Consider more realistic 2-D and 3-D fluctuations in a shock precursor.



- Original Parker instability
  - cosmic ray sources in disc
  - gravitational field of Galaxy
  - buoyant flux tubes inflate with cosmic rays and rise up

- Exactly the same physics should occur in a shock precursor
  - deceleration provides an effective gravitational field towards the shock
  - strong cosmic ray gradient away from shock
  - magnetic field loops linked to the shock should inflate and “rise up”.

Only way to avoid this effect would be to completely decouple diffusion from the density (and magnetic field!) distribution.

$$\frac{\partial \kappa}{\partial \rho} = 0$$

Thus it is impossible for the cosmic-ray pressure gradient to uniformly decelerate density fluctuations in the inflowing plasma and at the same time avoid inducing transverse velocity perturbations.

- Bell's non-resonant instability
  - Energetic cosmic rays penetrate upstream ignoring small-scale field structure - unmagnetised on these scales.
  - Return current of low-energy particles is forced through magnetised plasma
  - Field lines coil up and form helical structures around plasma voids.

- MHD is theory of strongly magnetised plasmas.
- Usually rewrite Lorentz force on plasma to eliminate currents using induction law.
- Need to modify this for case considered.

$$
\begin{aligned}
\nabla \wedge B &= j_{CR} + j_{th} \\
j_{th} &= \nabla \wedge B - j_{CR} \\
F &= j_{th} \wedge B \\
&= (\nabla \wedge B) \wedge B - j_{CR} \wedge B
\end{aligned}
$$

- If CR are strongly scattered then the additional force term can be shown to reduce to an additional cosmic ray pressure gradient driving both resonant wave growth and the acoustic instability

$$\langle j_{CR} \wedge B \rangle \approx \nabla P_{CR}$$

- But if they are not scattered, then produce current-driven instability.

$$\begin{aligned} j_{\mathrm{CR}} &\approx n_{CR} v_{\mathrm{drift}} \\ &\approx \frac{P_C}{pc} U \propto \rho U^3 \end{aligned}$$

Figure 4: The paths of four magnetic field lines in (x,y,z) at $t = 6$.

(from Bell 2005)

Good recent account and confirmation of Bell's magnetic field amplification process in arXiv0801.4486 by Zirakashvili, Ptuskin and Voelk.

Instability driven by the cosmic ray current causes spirals of magnetic field to expand and interact.

Turbulence and multiple local MHD shocks in precursor! Field saturates (Bell) at about

$$k\frac{B^2}{2\mu_0} \approx jB$$

$$\Longrightarrow \; B \propto j \propto \rho U^3$$

Nonlinear reaction effects:

- Shock precursors are almost certainly highly "turbulent" - not clear what implications if any this has for the modification theories.
- Easy to amplify small-scale magnetic field by stretching and twisting.
- Field can plausibly be increased by orders of magnitude, if not to equipartition (Bell predicts saturation a factor U/c below).

- Strong observational indications of amplified fields in young SNRs (with possible U^3 scaling according to J Vink).

- Allows acceleration of protons to “knee region” with ease - otherwise a bit difficult (but scale issue?).

- NB - upstream amplification is needed to reach higher energies, but the observational evidence is only for downstream fields!

Relativistic shocks

See arXiv:0807.3459 by Pelletier, Lemoine and Marcowith for a good recent account.

- In principle the same basic acceleration process, multiple shock crossings with magnetostatic scattering on either side, should work.
- But there are a number of major differences as well as at least one serious problem.

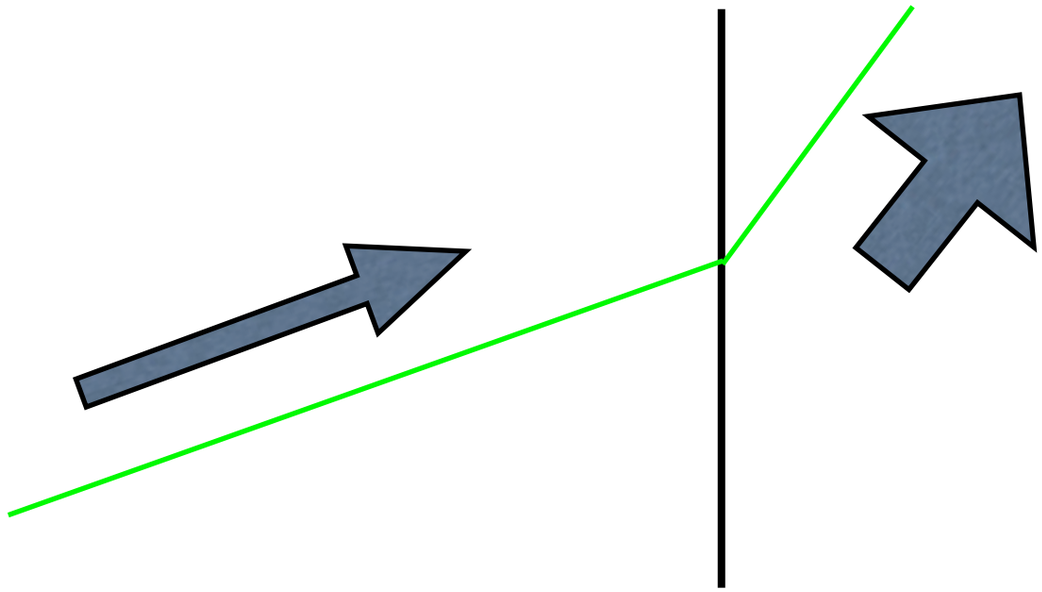
- Distributions are highly anistropic at relativistic shocks, and the energy changes are not small - relativistic shock acceleration is certainly not diffusive!

- Relativistic shocks with magnetic fields are generically superluminal - not clear that particles can in fact recross the shock if they are at all magnetised, or that they have time to be scattered in angle before being overtaken while upstream.

- If large scale regular field with well defined field lines, can divide shocks into those where the point of intersection between shock front and field line moves at less than the speed of light (sub-luminal shocks) and those where it moves faster (super-luminal shocks).

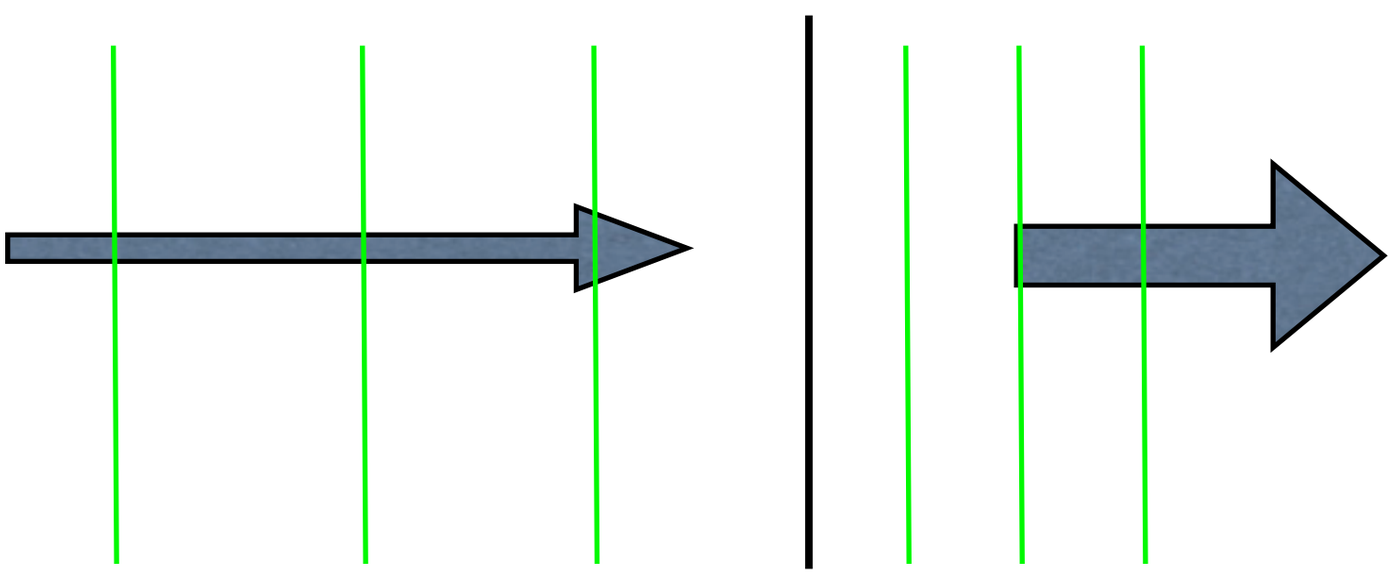
  - In sub-luminal case can boost into de Hoffman-Teller frame where this point is at rest.

  - In super-luminal case can boost to infinite velocity - ie field is strictly perpendicular.



Sub-luminal case

$$U \parallel B$$



Super-luminal case

$$U \perp B$$

- If particles are tied to field lines clear that superluminal shocks are not good sites for Fermi acceleration - requires fast cross-field diffusion and strong turbulence (or very random field).

- NB cross-field diffusion at perpendicular shocks can give Fermi acceleration at low speed non-relativistic shocks (Jokipii).

- Hard to see how this could work for strongly magnetised relativistic shocks though.

- However, if one ignores these issues and just assumes that a Fermi type acceleration occurs, then general agreement that a universal spectrum with exponent about 4.2 to 4.3 is formed.

- Recent development is very promising work by Anatoly Spitkovsky on pure electron positron shocks with self-generated Weibel fields where he sees direct evidence for Fermi acceleration in relativistic PIC simulations.

- Suggests (J Arons) that the solution to relativistic shock acceleration is that you need unmagnetised upstream media (or at least ones where the field is weak enough that the Weibel fields can dominate).

- However some form of relativistic shock acceleration is clearly needed for GRB models as well as pulsar wind nebulae.

Open Questions

- How do we describe turbulent precursors?
- Can PIC studies lead us to a quantitative model of injection?
- Why do we not clearly see shock precursors?
- Where are the Galactic pevatrons?
- Is non-standard transport important?
- Is acceleration at relativistic shocks similar, or qualitatitvely different?