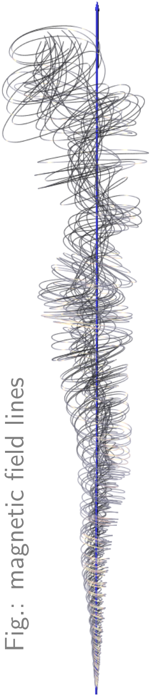

Fig.: magnetic field lines

Simulations of Kink Instabilities in Expanding Jets & Jets From Small-Scale Fields

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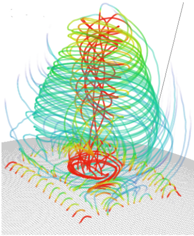
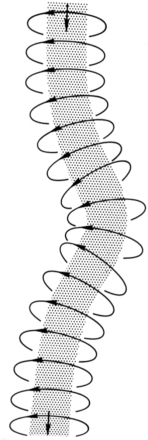
HEPRO II meeting, Buenos Aires
October 26, 2010



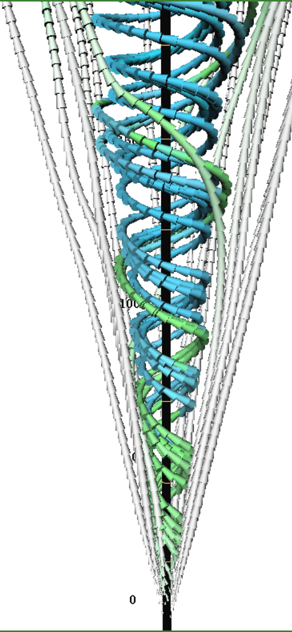
Kink instabilities: introduction and motivation



- driven by the free energy associated with B_φ, the field component responsible for the acceleration in the magnetic jet model
- grow on Alfvén crossing time scale (linear analyses)
- most studies (controlled fusion or jets) consider cylindrical configurations, but expansion important ...
- deformation of the jet, wiggles
- "disruption" through interaction with ambient medium ?
- induced field dissipation can lead to more acceleration (?) (Drenkhahn 2002; Giannios & Spruit 2006)
- are non-axisymmetric ⇒ full 3D simulations necessary

Kink instabilities in expanding jets

- expanding jets are especially prone:
  $B_{\mathrm{p}} \sim 1/R^2$ and $B_\varphi \sim 1/R$ in ballistic expansion
  (induction equation) $\Rightarrow B_\varphi$ tends to dominate
- $R \sim r^\alpha \Rightarrow \dfrac{\text{growth rate}}{\text{expansion rate}} \sim r^{1-\alpha}$
  - decollimation ($\alpha > 1$):
    instability loses importance with distance r
  - collimation ($\alpha < 1$):
    instability gains importance with distance r
  - conical ($\alpha = 1$):
    limiting case, depends on $v_R/v_{\mathrm{A}\varphi}$,
    no growth if $v_R \gtrsim v_{\mathrm{A}\varphi}$



[R . . . jet width (cylindrical radius),
r . . . jet height (spherical radius)]

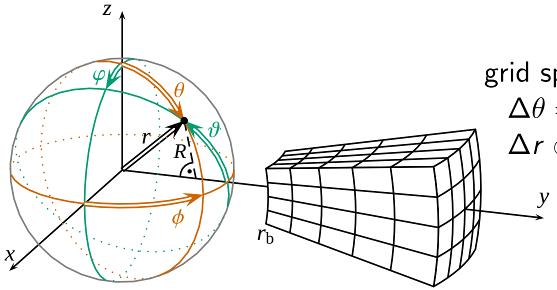
Simulation setup

Code: Eulerian, non-relativistic ideal MHD

Aim: simulate an expanding jet over a long distance in 3D

Grid: Spherical (r, θ, ϕ), with jet in equatorial direction

- computationally much more efficient than a Cartesian grid
- no coordinate singularities (as in polar direction)



grid spacing:

$\Delta\theta = \Delta\phi = \text{const}$ (uniform)

$\Delta r \propto r$ (logarithmic)

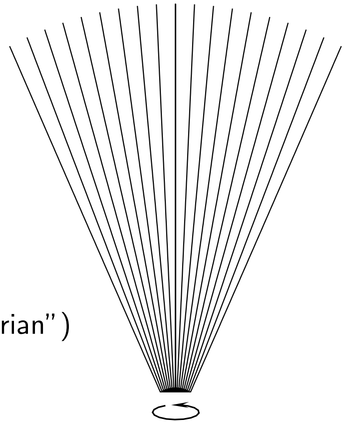
Simulation setup (1)

Initial conditions:

- monopole field $\mathbf{B}(r) \propto r^{-2}\hat{\mathbf{e}}_r$
- hydrostatic equilibrium with
  $\Phi \propto r^{-1}$ and $\beta = \text{const}$:
  $p \propto r^{-4}$, $\rho \propto r^{-3}$, $T \propto r^{-1}$, $v_\mathrm{A} \propto r^{-1/2}$

Boundary conditions:

- lower r (bottom, jet inlet):
  confined, rotating disk-like surface with
  $\Omega = \text{const}$ (rigid) or $\Omega \propto R^{-3/2}$ ("Keplerian")
- all other boundaries: open (outflow)



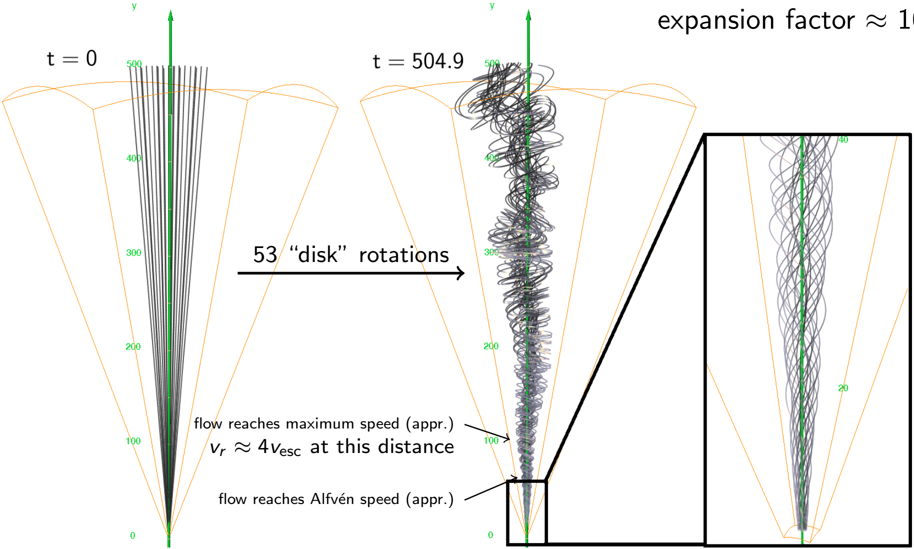
Scale-free units

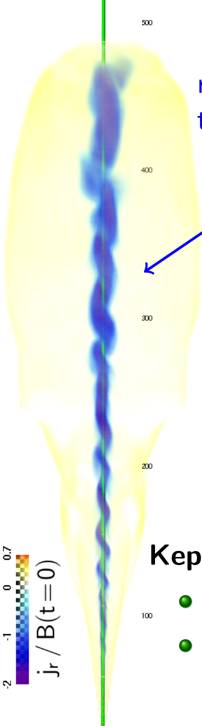
$\Rightarrow$ free parameters: plasma-β $= 1/9$, opening angle ϑ $= 5.7°$, $v_\varphi^{\max}$ at the inlet $= 0.33c_\mathrm{s} = 0.1v_{\mathrm{A}r}$

Comparison of axisymmetric 2.5D (stable) and 3D (unstable) simulations

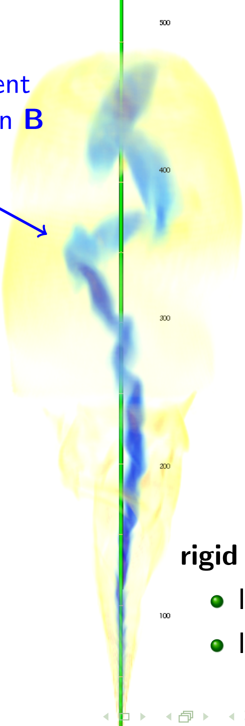
Wind-up of the magnetic field

$r = 5 \dots 505 \times$ initial diameter
expansion factor ≈ 100





radial backward current traces deformations in **B**

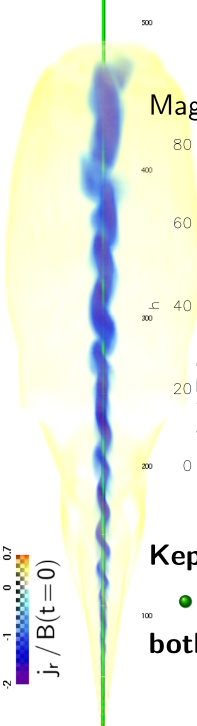


Keplerian rotation

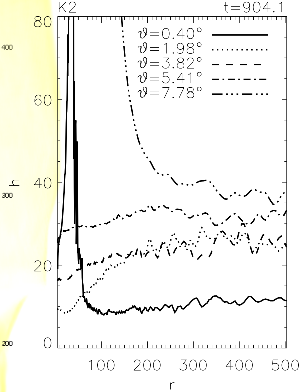
- small amplitude
- small wavelength

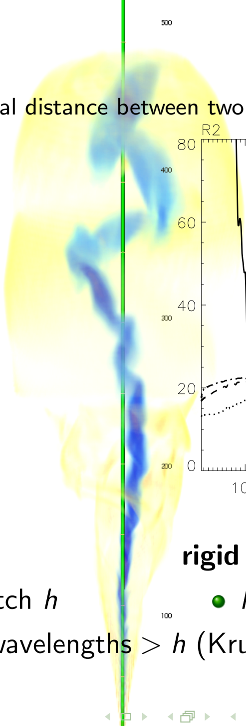
rigid rotation

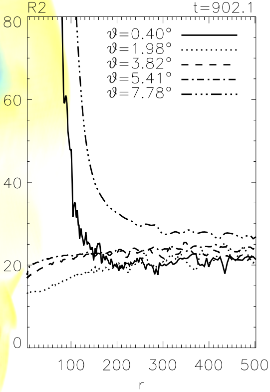
- large amplitude
- large wavelength



Magnetic pitch h = vertical distance between two field line windings







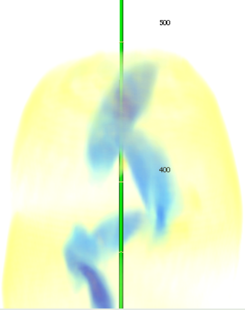
Keplerian rotation

- varying magnetic pitch h

rigid rotation

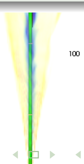
- $h \approx$ const with ϑ

both cases: instability wavelengths $> h$ (Kruskal–Shafranov)

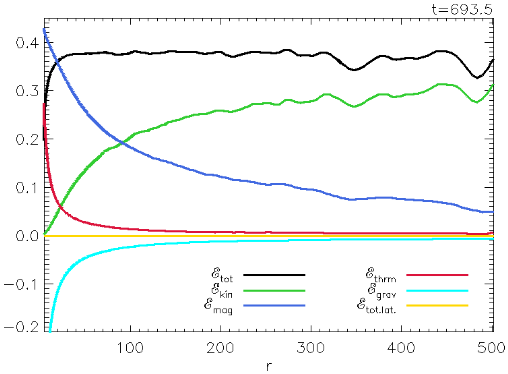


Further results:

- instabilities are comoving with the flow
- they grow initially exponentially in a comoving system, growth time $\sim$ Alfvén crossing time (consistent with linear theory)
- saturate at appr. 3° or run out of the box



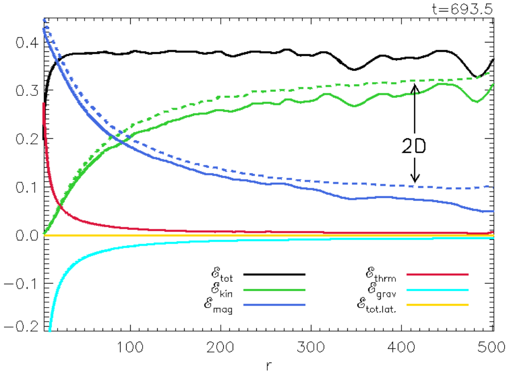
Energy flow rates [energy/time]



- quasi-stationary state is quickly attained
- conversion $\mathcal{E}_{\mathrm{mag}} \to \mathcal{E}_{\mathrm{kin}}$ rather efficient
- no significant dissipation of magnetic enthalpy $\mathcal{E}_{\mathrm{mag}}$

$$\mathcal{E}_{\mathrm{tot}}(t,r) = \int_{r=\mathrm{const}} \Big(\underbrace{\frac{1}{2}\rho v^2 v_r}_{\text{kinetic}} + \underbrace{\frac{\gamma}{\gamma-1} p v_r}_{\text{thrm. enthalpy}} + \underbrace{\rho \Phi v_r}_{\text{gr. potential}} + \underbrace{S_r}_{\text{magn. enthalpy}} \Big) \mathrm{d}A$$

Energy flow rates [energy/time]



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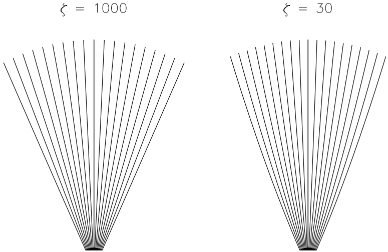
New setup: parabolically shaped initial field



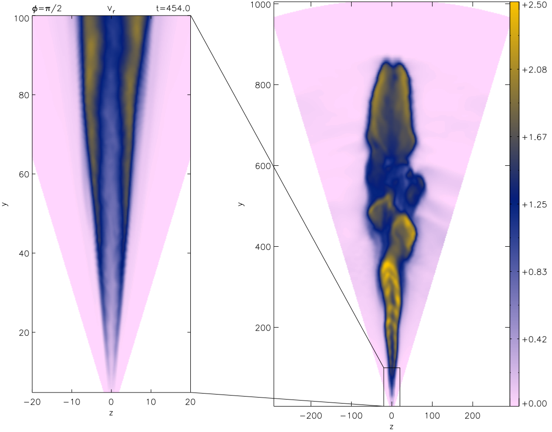
≈monopole mixed parabolic

(Cao & Spruit 1994)

- generates more collimated jets, promoting instability
- still a potential field (force-free)

Simulation with parabolic field and dissipation

- simulated distance: $1000 \times$ initial jet diameter, $\approx 15 \times$ (min.) Alfvén radius
- expansion factor ≈ 200
- ambient medium: $\rho = 1 \ldots 10^{-7}$



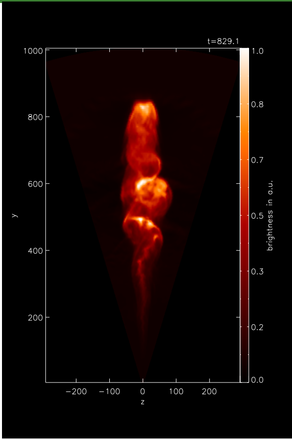
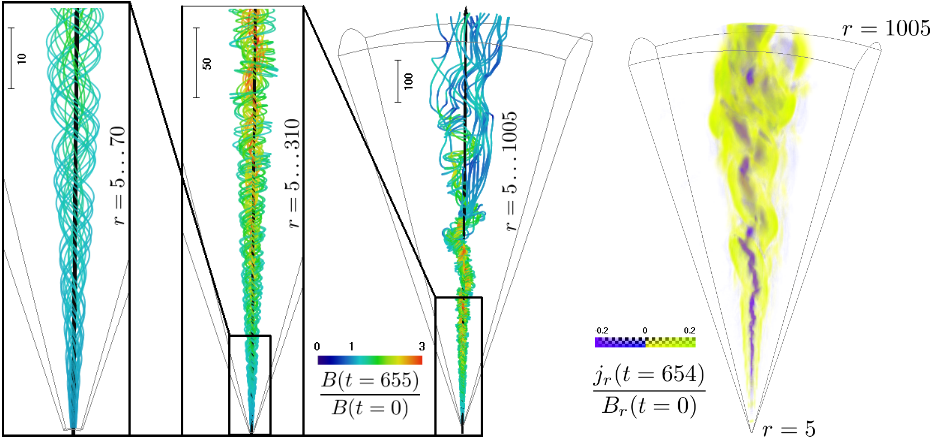


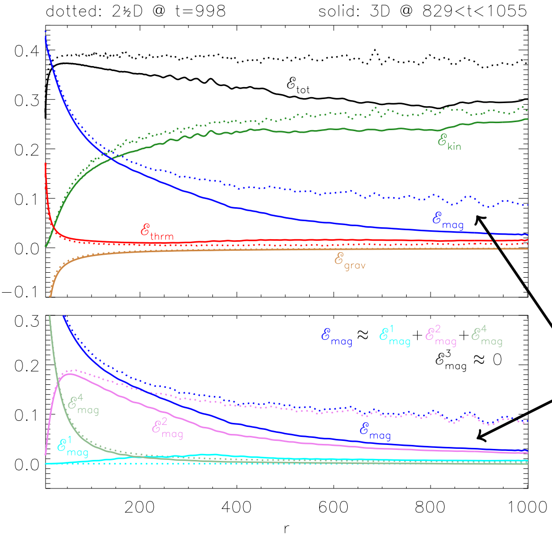
Fig.: Volume rendering with source $= T$ and opacity $= \rho$, relative to environment

Magnetic field & current density



- magnetic field evolution: ordered, helical $\rightarrow$ strong B_φ $\rightarrow$ instabilities, dissipation of B_φ $\rightarrow$ predominantly poloidal
- current density $(\nabla \times \mathbf{B})_r$ shows "disruption" of the magnetic field structure by instabilities

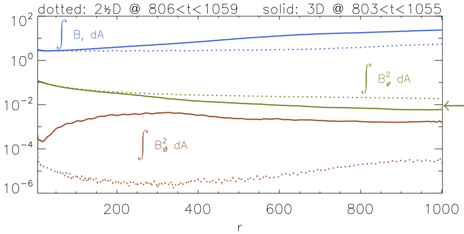
Energy flow rates



$$\mathcal{E} = \int_{r=\mathrm{const}} \overbrace{\left(\frac{\rho v^2}{2} + \frac{\gamma p}{\gamma - 1} + \rho\Phi\right) + \underbrace{S_r}_{B_\vartheta^2 v_r + B_\varphi^2 v_r - B_\vartheta B_r v_\vartheta - B_\varphi B_r v_\varphi}}^{\text{flux}} \, \mathrm{d}A$$

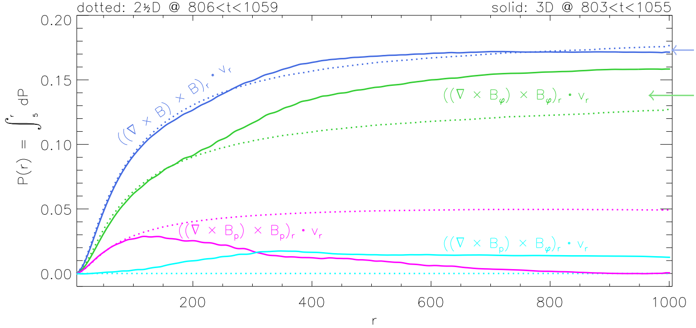
- with instability: significant reduction of magnetic enthalpy (Poynting flux)
- kinetic energy not increasing despite dissipation of B_φ

Steepening of ∇B_φ^2, but no net accelerative force



B_φ^2 dissipates

Fig.: Magnetic field components integrated over the jet cross section



total Lorentz force delivers the same power

Lorentz force associated with B_φ delivers more power

Fig.: Rate of work done by the components of the Lorentz force in r-direction

Poloidal flux distribution: “diverging nozzle” effect?!

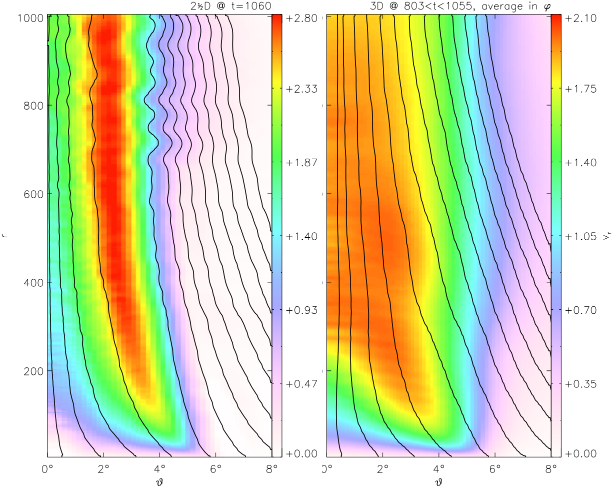


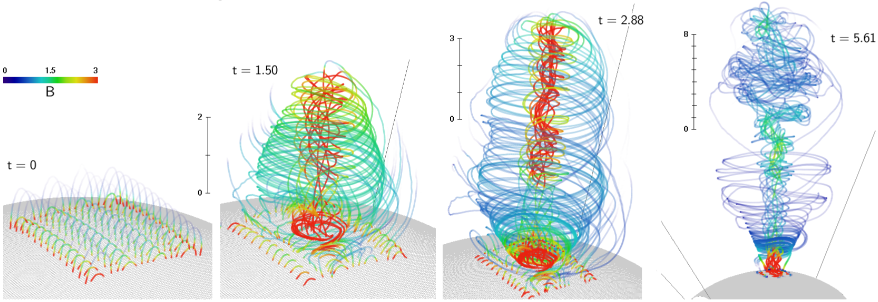
Fig.: poloidal field lines
+ velocity
(vertical line means
radial field)

2.5D (left): magnetic surfaces diverge more rapidly for some angles
→ more efficient acceleration (“magnetic nozzle” effect, Begelman & Li 1994)

Magnetic jet models usually assume an ordered, axially symmetric large-scale field of uniform polarity:

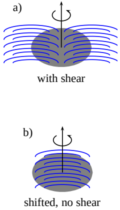
- too large for MRI turbulence (max. scale = disk thickness)
- not easily trapped by an accretion disk, tends to diffuse outwards (e.g. van Ballegooijen 1989)

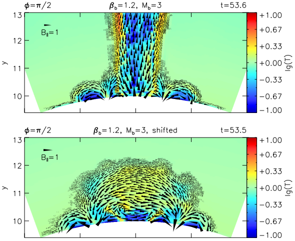
Are small-scale magnetic loops a viable alternative?



Requirements for jets from small-scale fields

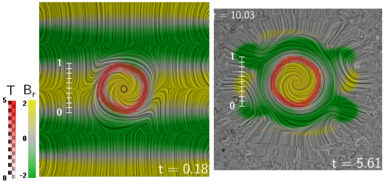
- magnetic shear through differential rotation of the loops' footpoints



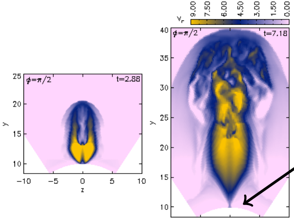


else: production of B_φ not efficient enough for magnetic acceleration/collimation

- continuous replenishment of the loops; otherwise: decay of the "stirred" field at the base, jet is only transient

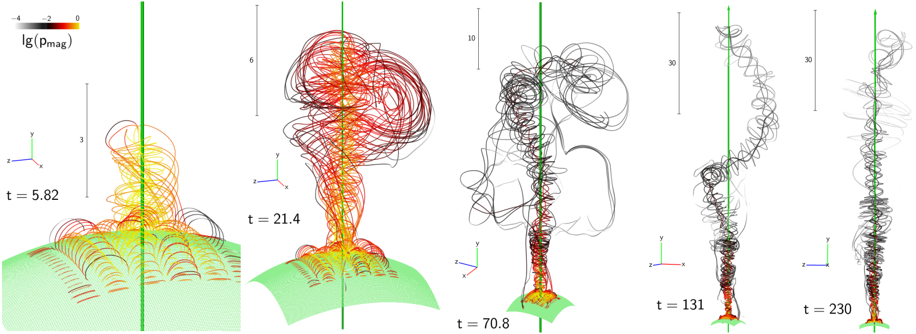


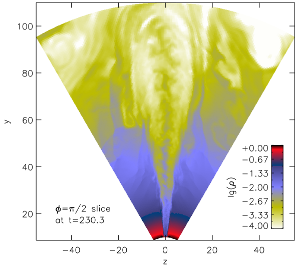
stirred field at the base dissipates



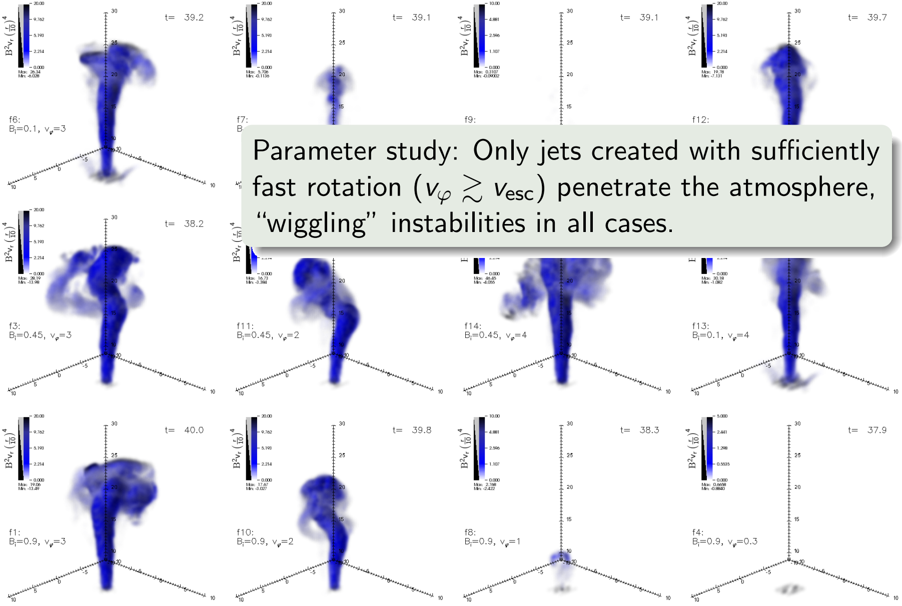
jet is "choked off"

A long simulation with constant inflow of magnetic loops





- long (upper bnd. at 100 times source size)
- well-collimated
- occasional kinks
- appr. 75% of the Poynting flux is converted to kinetic energy



Parameter study: Only jets created with sufficiently fast rotation ($v_\varphi \gtrsim v_{esc}$) penetrate the atmosphere, "wiggling" instabilities in all cases.

Summary

- Using a spherical, logarithmically spaced grid we cover 3 orders of magnitude in length scales (jet length / initial diameter).
- Magnetic fields become distorted by kink instabilities,
  jet is not "disrupted" but decay of the toroidal magnetic field
- Magnetic $\rightarrow$ kinetic energy conversion is fairly efficient ($\sim$80%)
- Collimation ($\mathrm{d}\vartheta/\mathrm{d}r < 0$) aggravates instabilities,
  significant dissipation of B_φ
  - the flow is not accelerated over the stable case (2.5D),
    less favorable distribution of the poloidal magnetic flux
  - eradiated magnetic field energy $\rightarrow$ "knots" like in protostellar jets
- Long jets may as well arise from small-scale fields if there is ample twist through differential rotation of the footpoints.

See also: Moll et al. 2008, A&A 492 pp. 612–630, arXiv:0809.3165
Moll 2008, A&A in press, arXiv:0809.316